

# One-Way Arbitrage and Its Implications for the Foreign Exchange Markets

*Alan V. Deardorff*  
*Gerald R. Ford School of Public Policy*  
*University of Michigan*

*For presentation at Beijing Institute of Technology*  
*June 8, 2012, Beijing*

# The Issue

- How closely are spot and forward exchange markets tied together?
  - The standard answer is based on “covered interest arbitrage” (CIA)
  - I argue that, in the presence of small transactions costs,
    - Exchange rates will be tied closer together than that form of arbitrage can explain
    - “One-way arbitrage” (OWA) instead will constrain the rates
    - Covered interest arbitrage will never occur

# The Source

Alan V. Deardorff, "One-Way Arbitrage and Its Implications for the Foreign Exchange Markets," *Journal of Political Economy* 87(2), April 1979, pp. 351-364.

# Conventional versus One-way Arbitrage

- In conventional arbitrage, an agent from outside of two or more markets exploits price differences to make a profit.
- In one-way arbitrage, a buyer or seller from inside these markets chooses among them to minimize cost.

# Conventional versus One-way Arbitrage

- Example: Price of tomatoes in two markets,  $P_1$  and  $P_2$ . Suppose  $P_1 < P_2$ 
  - Conventional arbitrage would have an entrepreneur buy tomatoes in market 1 at price  $P_1$  and sell them in market 2 at price  $P_2$ . That would raise  $P_1$  and lower  $P_2$ .
  - One-way arbitrage would have buyers of tomatoes just buy from the lower-priced market. That would prevent any sales at the higher price.

# Conventional versus One-way Arbitrage

- Without transactions costs, implications are the same:  $P_1 = P_2$ .
- With transactions costs, implications are different
  - Conventional arbitrageur incurs 2 costs, one buying and one selling
  - One-way arbitrageur incurs only 1 cost, and it may be the same for both markets, making it irrelevant
  - Thus one-way arbitrage will drive prices closer together than conventional arbitrage

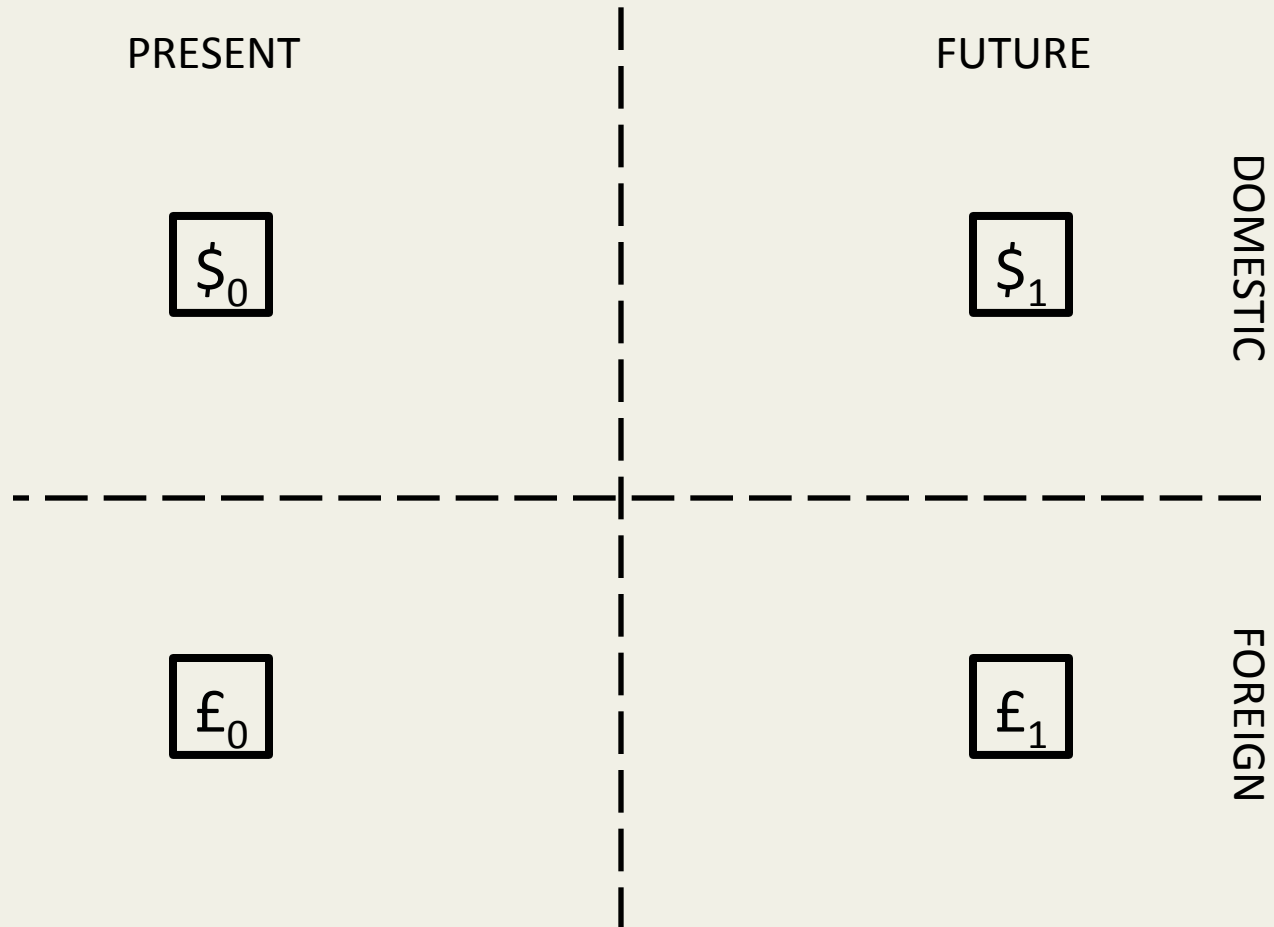
# Conventional versus One-way Arbitrage

- My story is just this one, but told in the more complex world of four markets:
  - Spot market
  - Forward market
  - Domestic securities market
  - Foreign securities market

} For exchanging domestic and foreign currencies

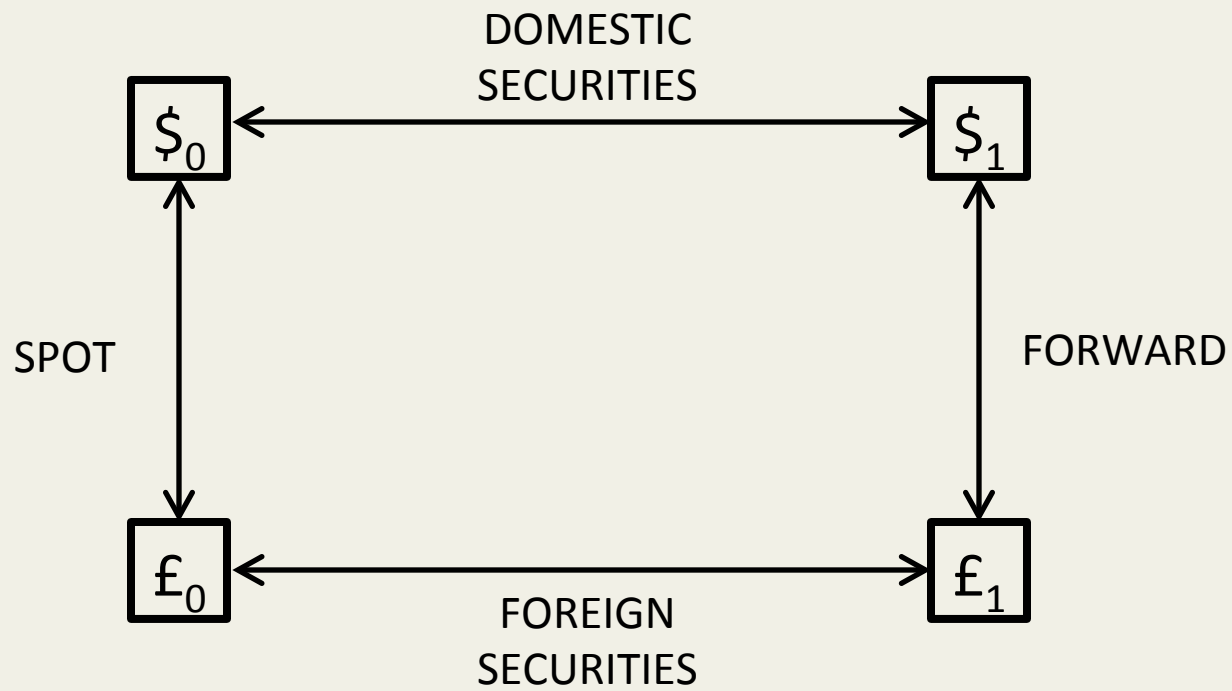
} For borrowing and lending
- What these markets trade are really four goods:
  - Domestic money now
  - Foreign money now
  - Domestic money in the future
  - Foreign money in the future

# The "Goods"

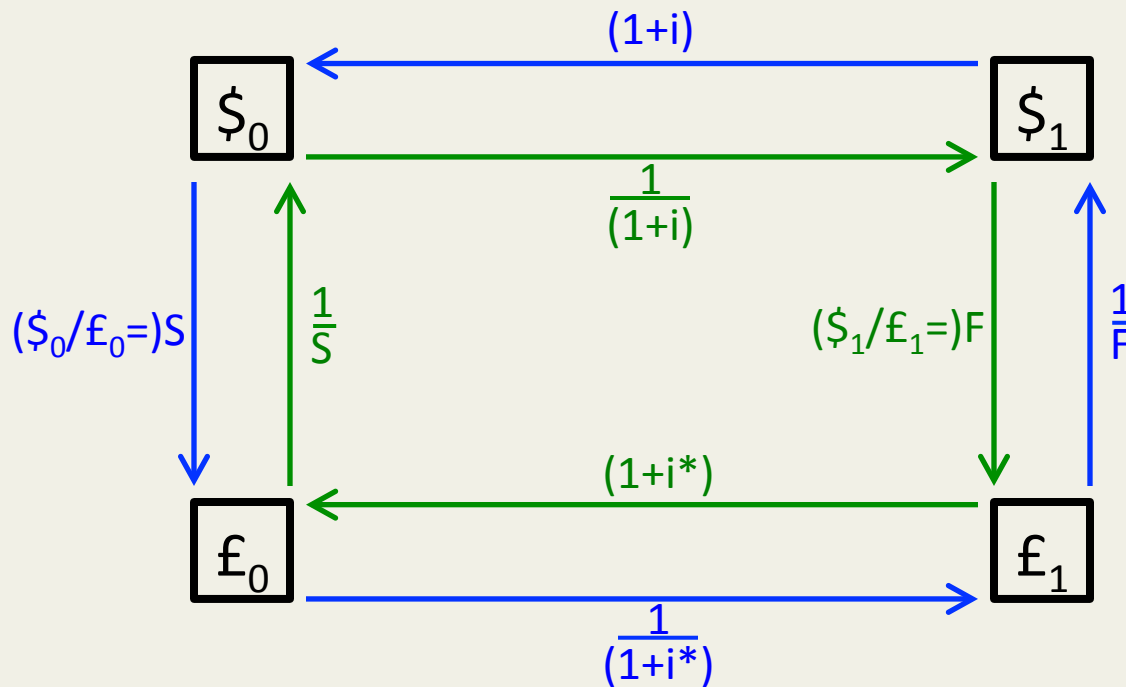




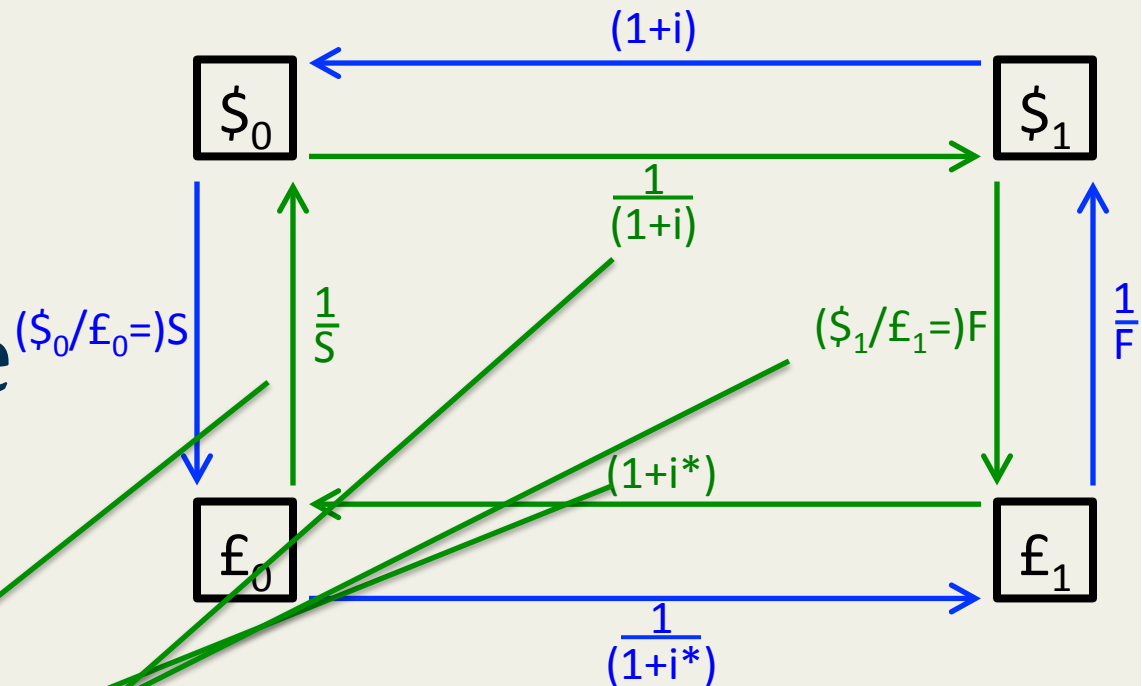
# The Markets



# The Rates (without t-costs)



# Covered Interest Arbitrage (without t-costs)



$$C(\$_0\pounds_1) = \frac{1}{S} (1+i^*) F \frac{1}{(1+i)} = \frac{F}{S} \frac{(1+i^*)}{(1+i)} \geq 1$$

Otherwise profit > 0 from clockwise path.

$$C(\$_0\pounds_1) = (1+i) \frac{1}{F} \frac{1}{(1+i^*)} S = \frac{S}{F} \frac{(1+i)}{(1+i^*)} \geq 1$$

Otherwise profit > 0 from counter-clockwise path.

$$\rightarrow \frac{F}{S} \frac{(1+i^*)}{(1+i)} = 1 \rightarrow F = \boxed{F_0 = S \frac{(1+i)}{(1+i^*)}}$$

The "Interest Parity Value"

# The Forward Premium (without t-costs)

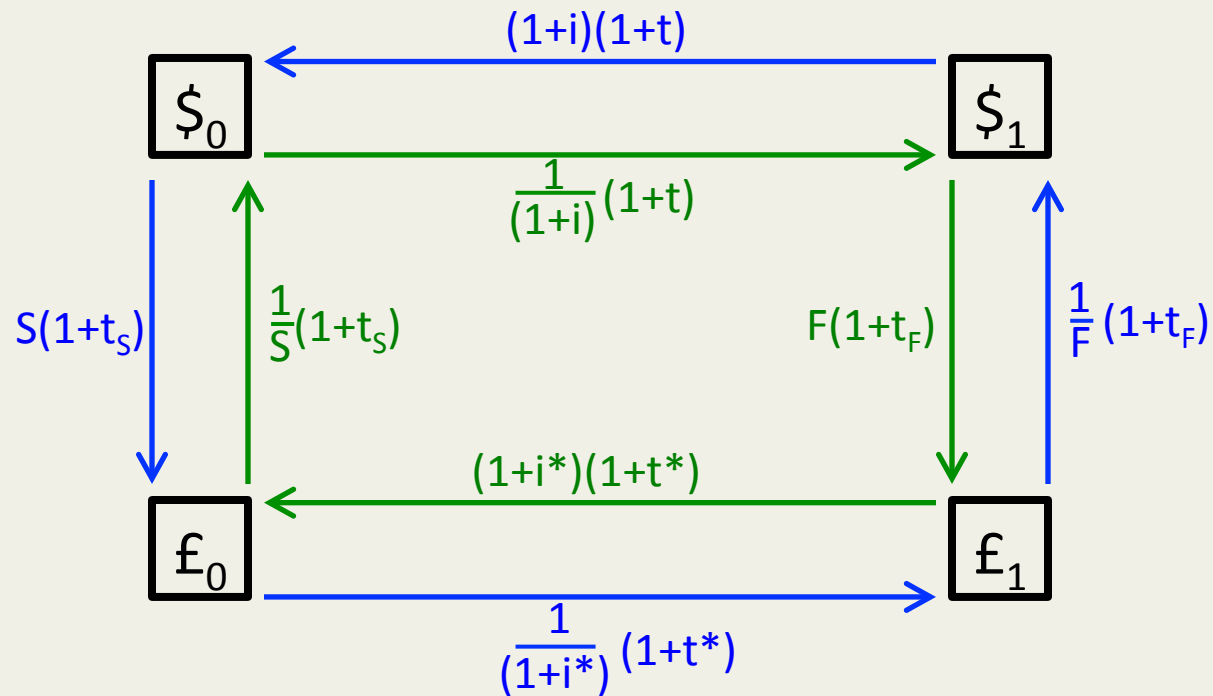
- The Forward Premium is the percentage by which the forward rate exceeds the spot rate.

$$P = \frac{F-S}{S}$$

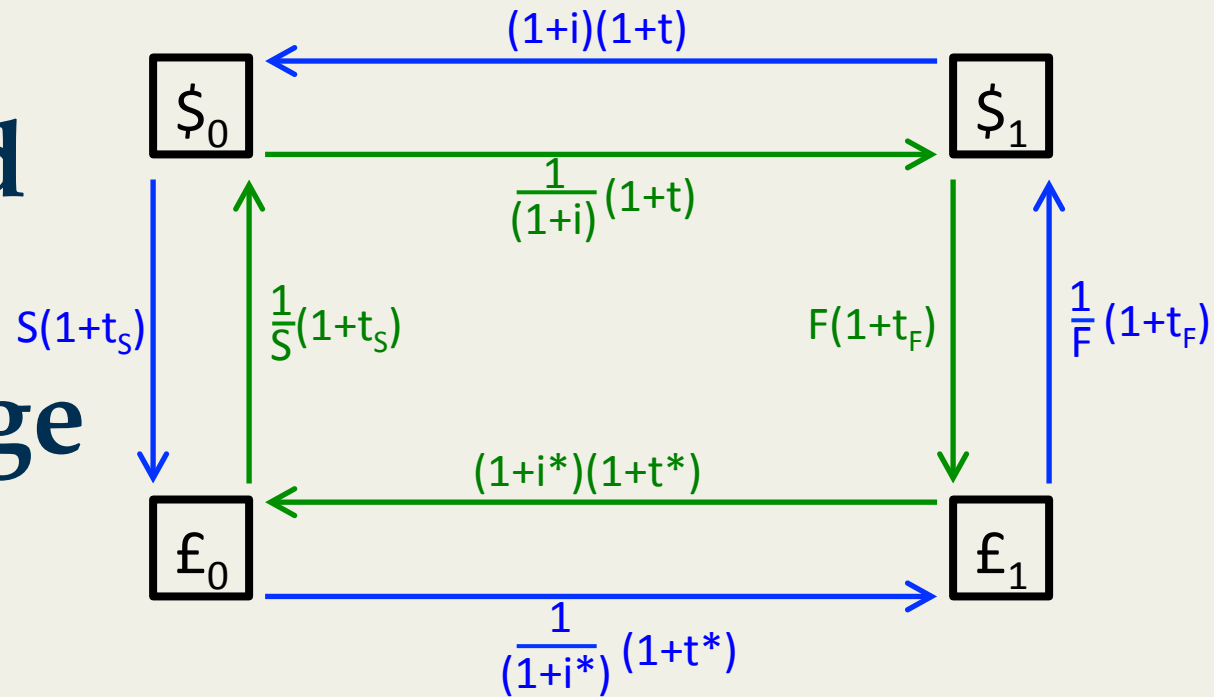
- Thus

$$P_0 = \frac{F_0 - S}{S} = \frac{(1+i)}{(1+i^*)} - 1 = \frac{(1+i) - (1+i^*)}{(1+i^*)} \approx i - i^*$$

# The Rates (with t-costs)



# Covered Interest Arbitrage (with t-costs)



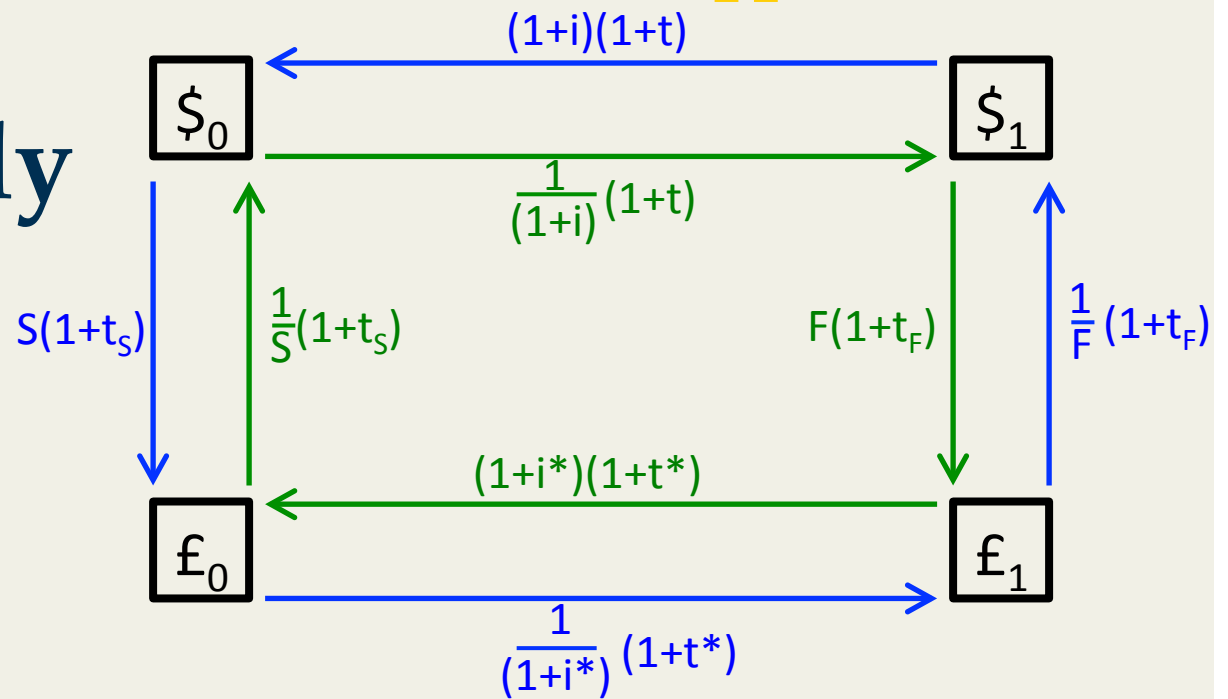
$$C(\$01) = \frac{1}{S}(1+t_S)(1+i^*)(1+t^*)F(1+t_F)\frac{1}{(1+i)}(1+t)$$

$$= \frac{F(1+i^*)}{S(1+i)}(1+t)(1+t^*)(1+t_S)(1+t_F) \geq 1$$

Thus

$$\frac{F}{F_0} \geq \frac{1}{(1+t)(1+t^*)(1+t_S)(1+t_F)}$$

Similarly



$$C(\$_0 1) = (1+i)(1+t) \frac{1}{F} (1+t_F) \frac{1}{(1+i^*)} (1+t^*) S(1+t_S)$$

$$= \frac{S}{F} \frac{(1+i)}{(1+i^*)} (1+t)(1+t^*)(1+t_S)(1+t_F) \geq 1$$

Thus 
$$\frac{F}{F_0} \leq (1+t)(1+t^*)(1+t_S)(1+t_F)$$

- Combining the limits due to both directions of CI arbitrage:

$$\frac{1}{(1+t)(1+t^*)(1+t_S)(1+t_F)} \leq \frac{F}{F_0} \leq (1+t)(1+t^*)(1+t_S)(1+t_F)$$

- Note that  $(1+x)(1+y) \approx 1+x+y$  &  $1/(1+x) \approx 1-x$  when  $x, y$  are small (as  $t, t^*$ , etc. are).

Then

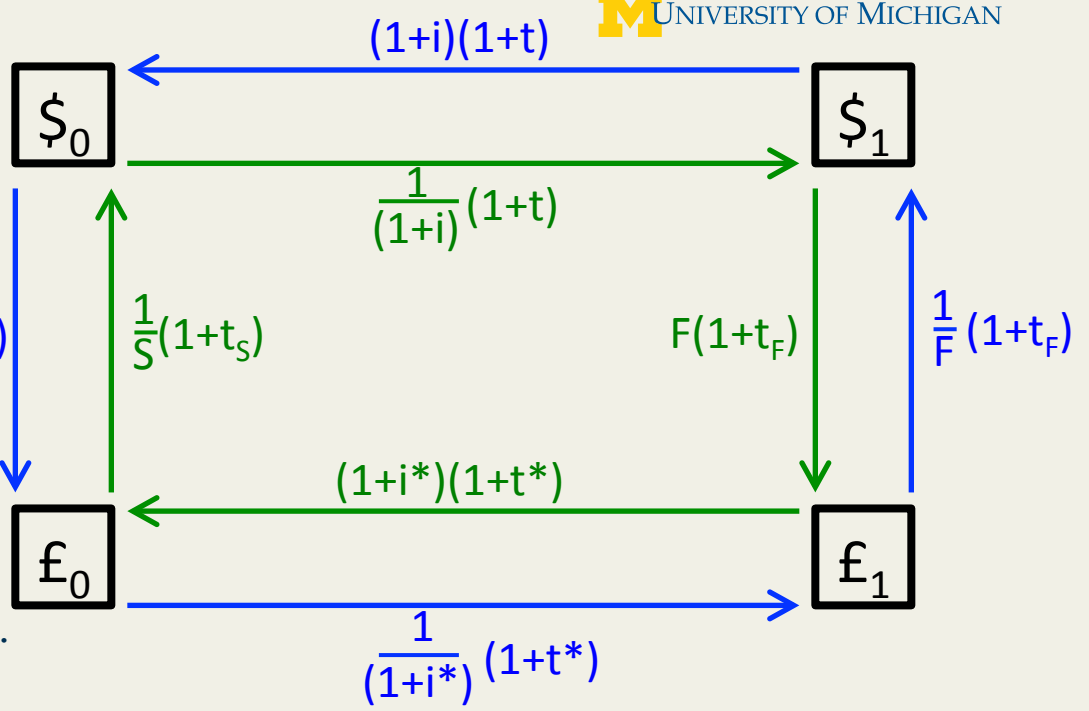
$$\left| \frac{F - F_0}{F_0} \right| \leq t + t^* + t_S + t_F \quad \text{Limits due to CIA}$$

- Thus the divergence of  $F$  from  $F_0$  is bounded by the sum of the four transactions costs.



# One-Way Arbitrage

Consider a trader who wants to exchange  $\$_0$  for  $\pounds_0$ .



- They can buy  $\pounds_0$  in the spot market for cost  $S(1+t_s)$
  - Or they can achieve the same result by
    - Lending dollars domestic,
    - Buying pounds forward, and
    - Borrowing pounds abroad.
- OWA Cost:  
 $\frac{1}{(1+i)}(1+t) F(1+t_f) (1+i^*)(1+t^*)$
- If OWA Cost < direct cost, then nobody will demand pounds on the spot market!

# Implication of OWA

- If supply in the spot market is positive, then equilibrium requires that demanders not prefer OWA, and therefore that

$$S(1+t_s) \leq \frac{1}{(1+i)} (1+t) F(1+t_f) (1+i^*)(1+t^*)$$

or

$$\frac{(1+t_s)}{(1+t)(1+t^*)(1+t_f)} \leq \frac{F(1+i^*)}{S(1+i)}$$

Using  $F_0$  and the approximations, this is

$$\frac{F}{F_0} \geq 1 + t_s - t - t^* - t_f$$

# Implications of OWA

- That may not seem likely to be binding. But if we repeat this sort of analysis for suppliers and demanders of both spot and forward pounds, we get four conditions, all of which must hold in order for both spot and forward markets to be used.
- These are shown in TABLE 1 of the paper.

TABLE 1  
CONDITIONS FOR DIRECT MARKET TRANSACTIONS

(C1) If demanders of current pounds use the spot market, then

$$-\frac{F - F_0}{F_0} \leq (t + t^*) + (t_F - t_S).$$

(C2) If demanders of future pounds use the forward market, then

$$\frac{F - F_0}{F_0} \leq (t + t^*) - (t_F - t_S).$$

(C3) If suppliers of current pounds use the spot market, then

$$\frac{F - F_0}{F_0} \leq (t + t^*) + (t_F - t_S).$$

(C4) If suppliers of future pounds use the forward market, then

$$-\frac{F - F_0}{F_0} \leq (t + t^*) - (t_F - t_S).$$

**NOTE ERROR:**  
Should have a  
minus sign




TABLE 1

CONDITIONS FOR DIRECT MARKET TRANSACTIONS

(C1) If demanders of current pounds use the spot market, then

$$\text{Spot Demand} > 0: -\frac{F - F_0}{F_0} \leq (t + t^*) + (t_F - t_S).$$

(C2) If demanders of future pounds use the forward market, then

$$\text{Future Demand} > 0: \frac{F - F_0}{F_0} \leq (t + t^*) - (t_F - t_S).$$

(C3) If suppliers of current pounds use the spot market, then

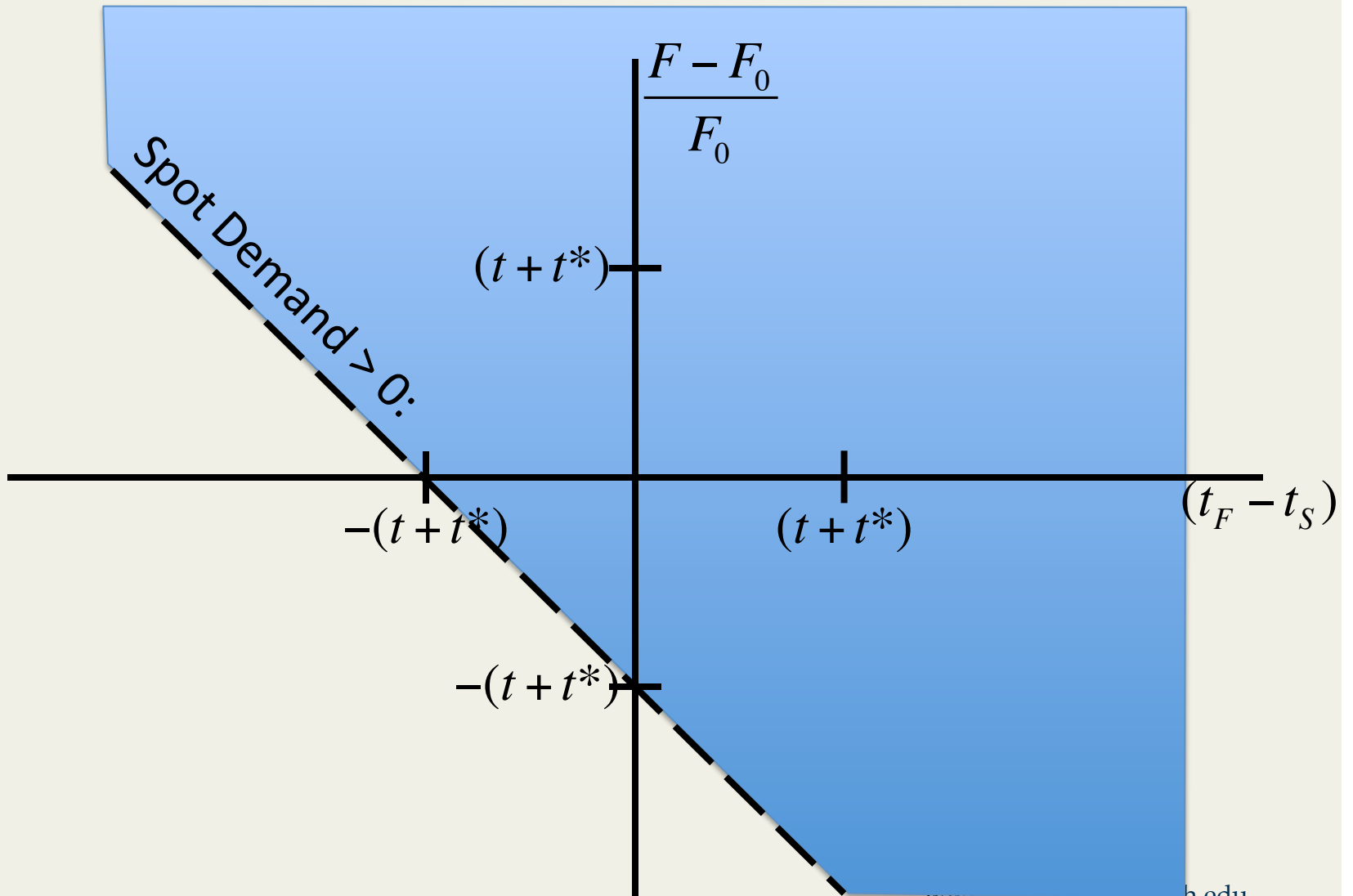
$$\text{Spot Supply} > 0: \frac{F - F_0}{F_0} \leq (t + t^*) + (t_F - t_S).$$

(C4) If suppliers of future pounds use the forward market, then

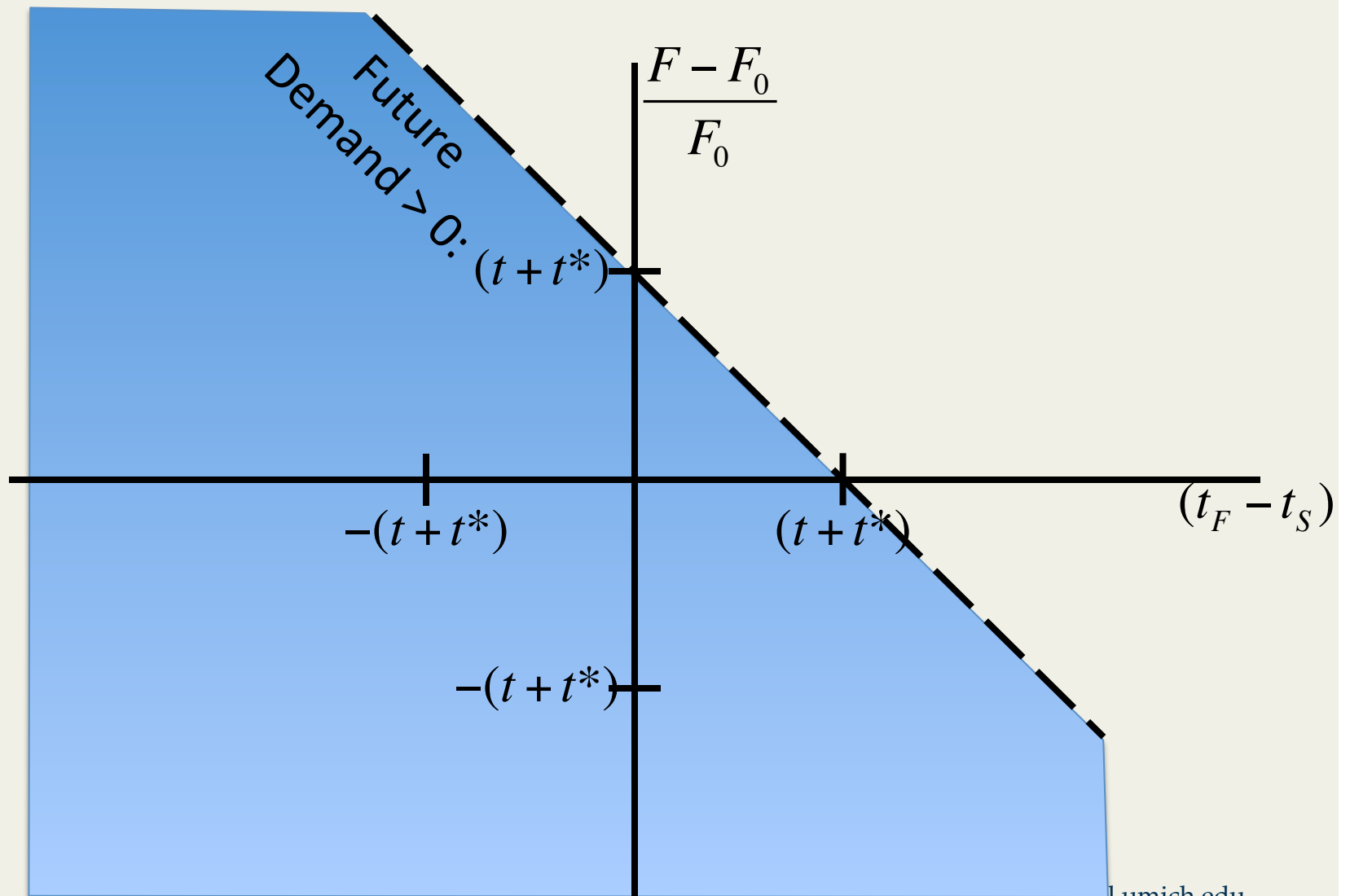
$$\text{Future Supply} > 0: -\frac{F - F_0}{F_0} \leq (t + t^*) - (t_F - t_S).$$

**These constrain where demands and supplies are positive, and thus where equilibrium is possible.**

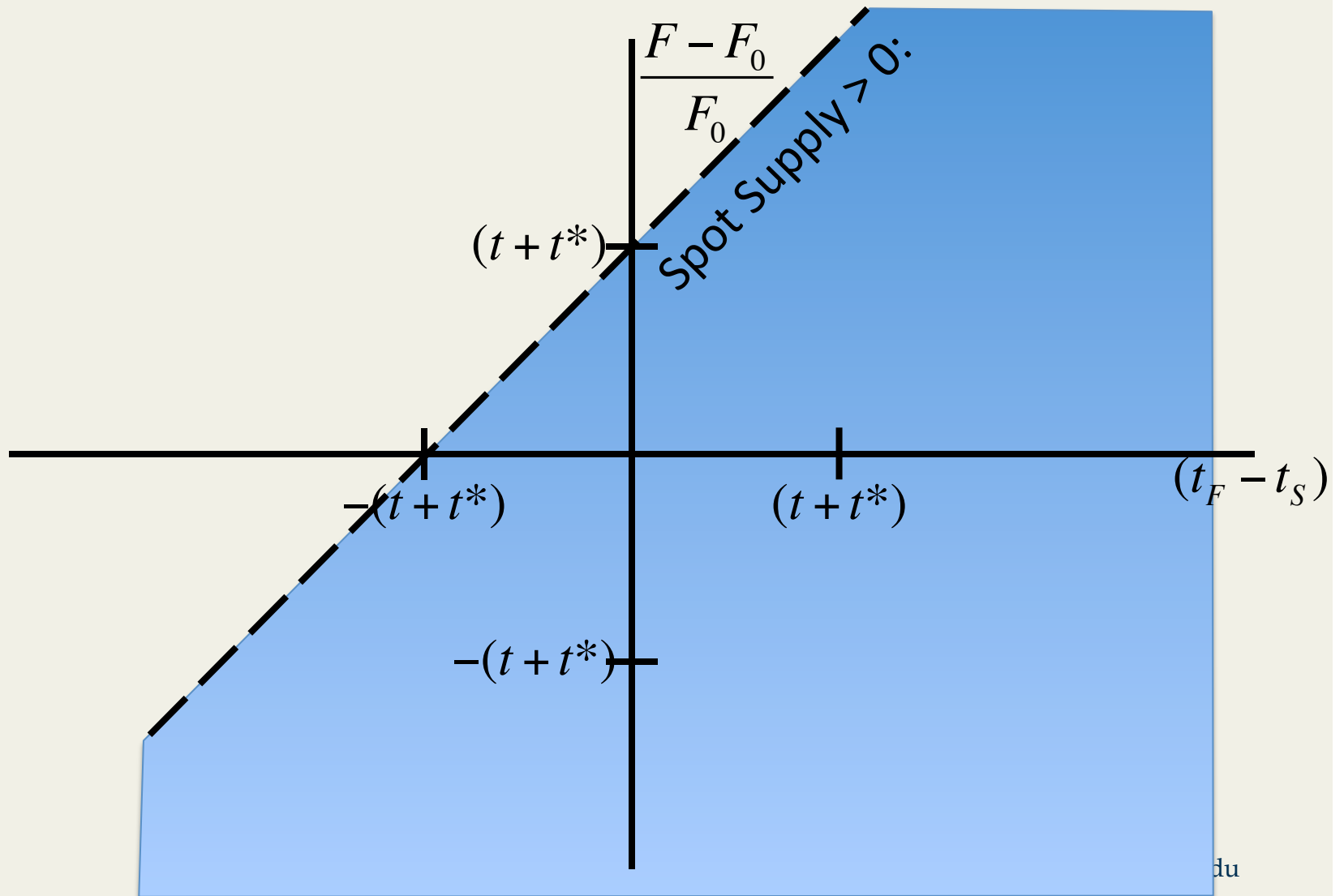
Spot Demand > 0: 
$$-\frac{F - F_0}{F_0} \leq (t + t^*) + (t_F - t_S)$$



Future Demand > 0: 
$$\frac{F - F_0}{F_0} \leq (t + t^*) - (t_F - t_S)$$

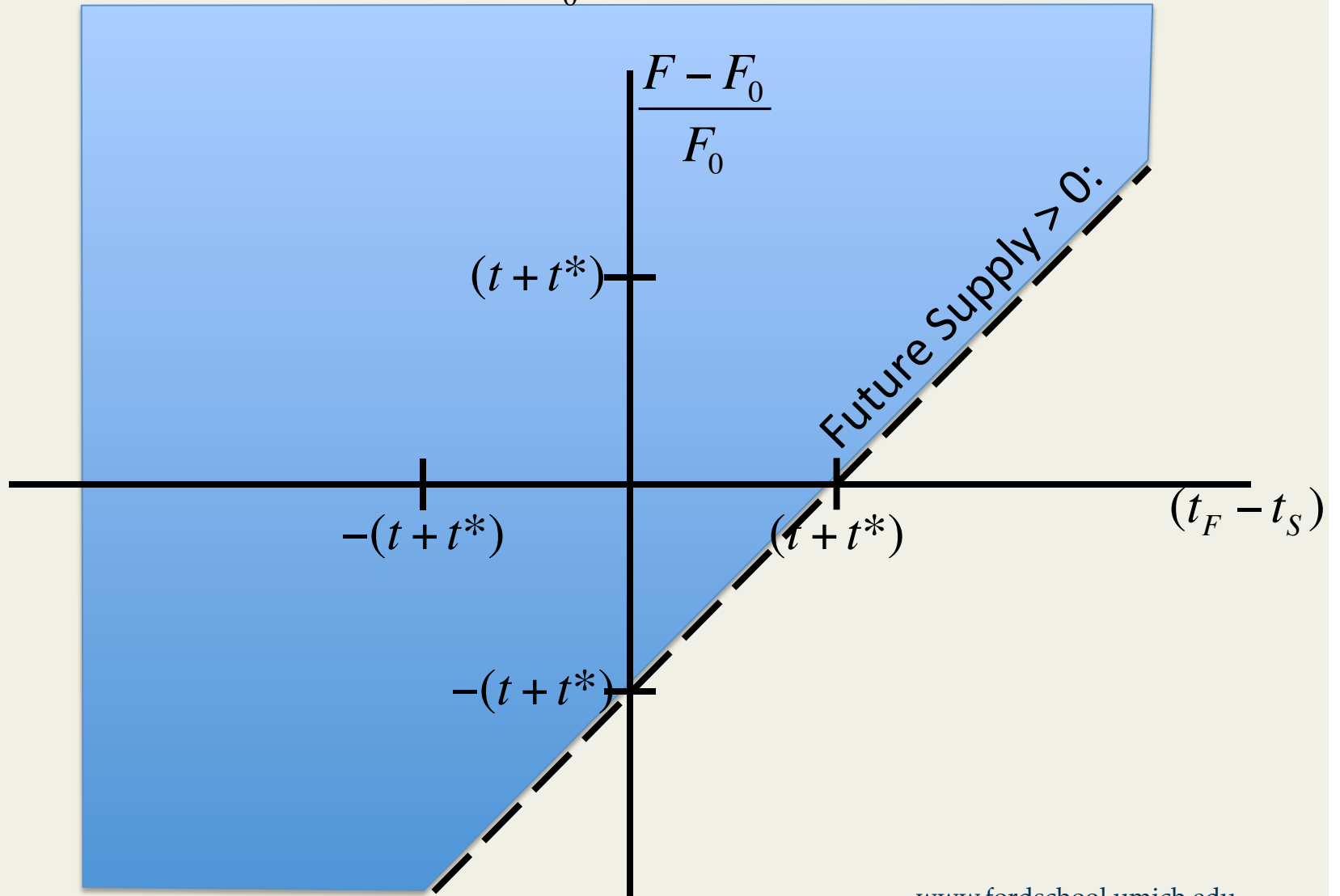


Spot Supply > 0: 
$$\frac{F - F_0}{F_0} \leq (t + t^*) + (t_F - t_S)$$

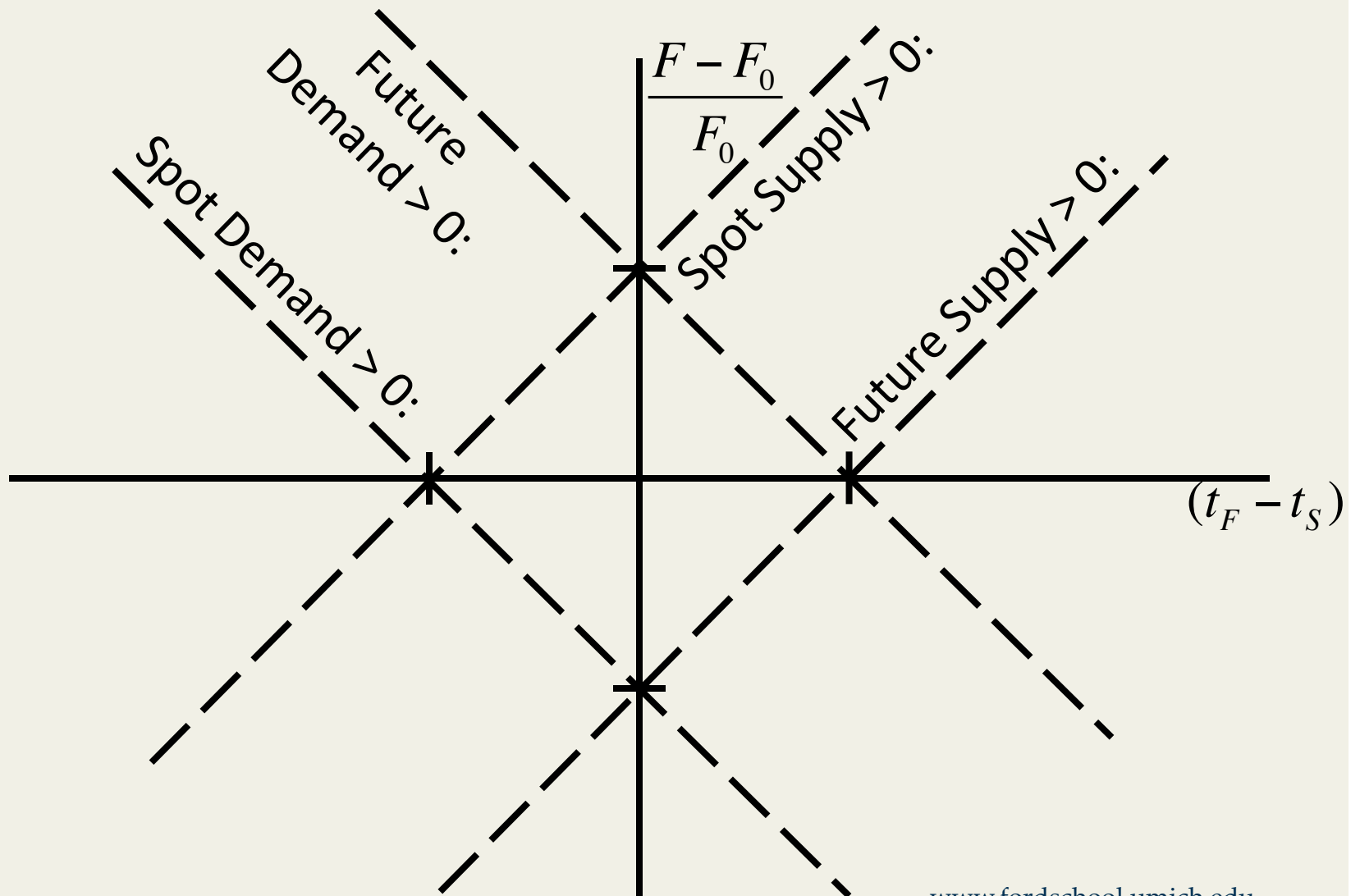




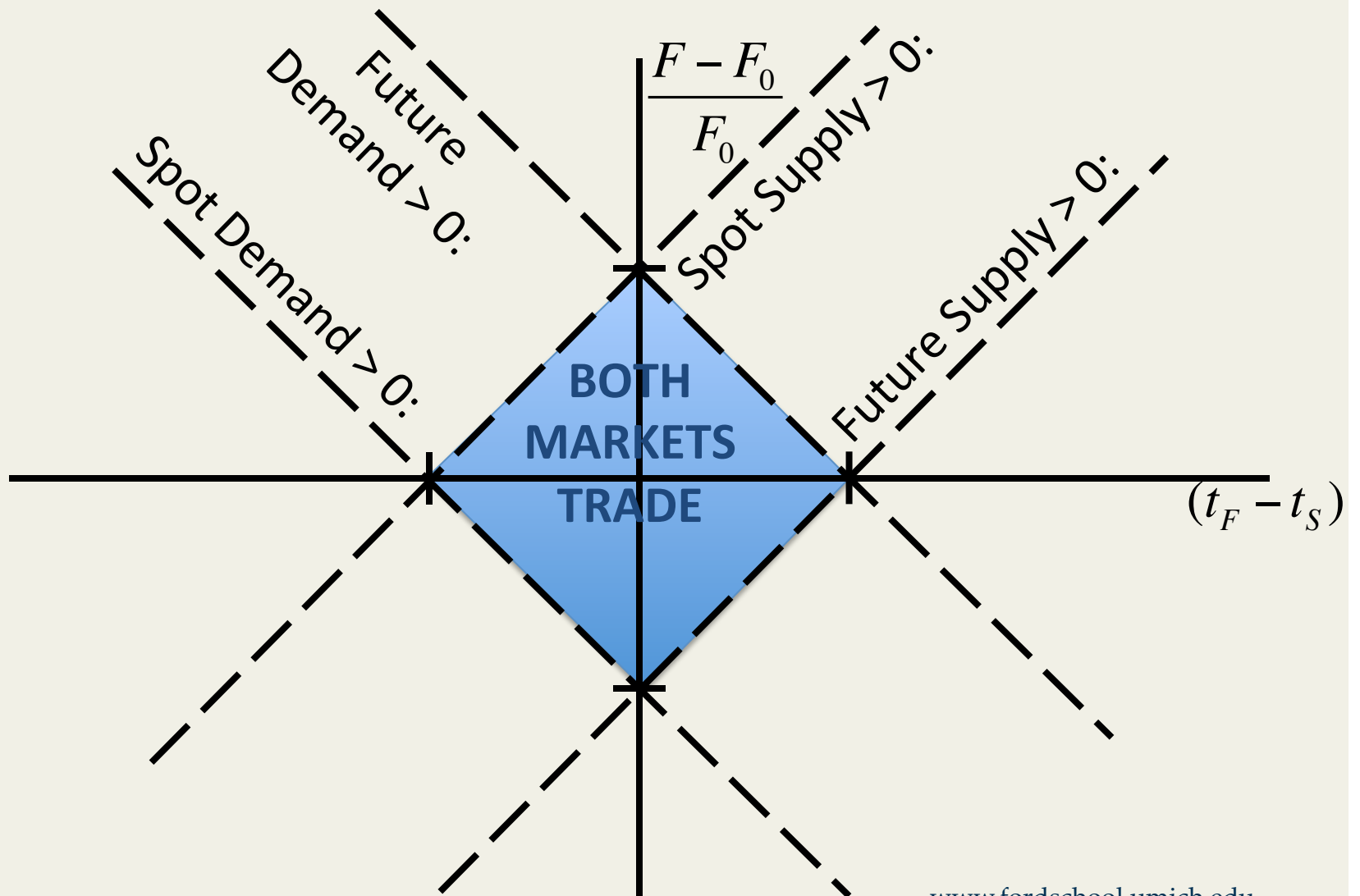
Future Supply > 0: 
$$-\frac{F - F_0}{F_0} \leq (t + t^*) - (t_F - t_S)$$



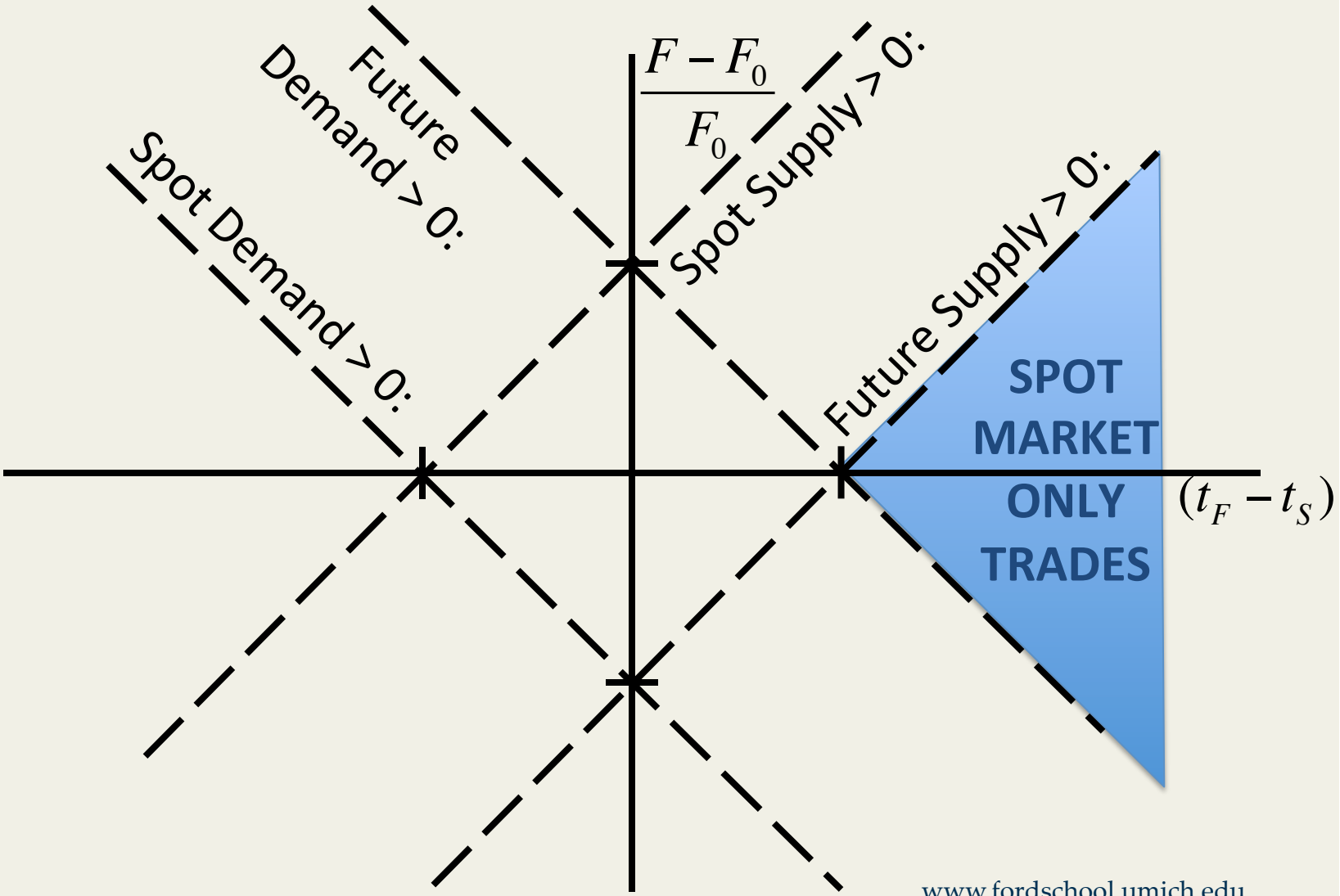
In combination:



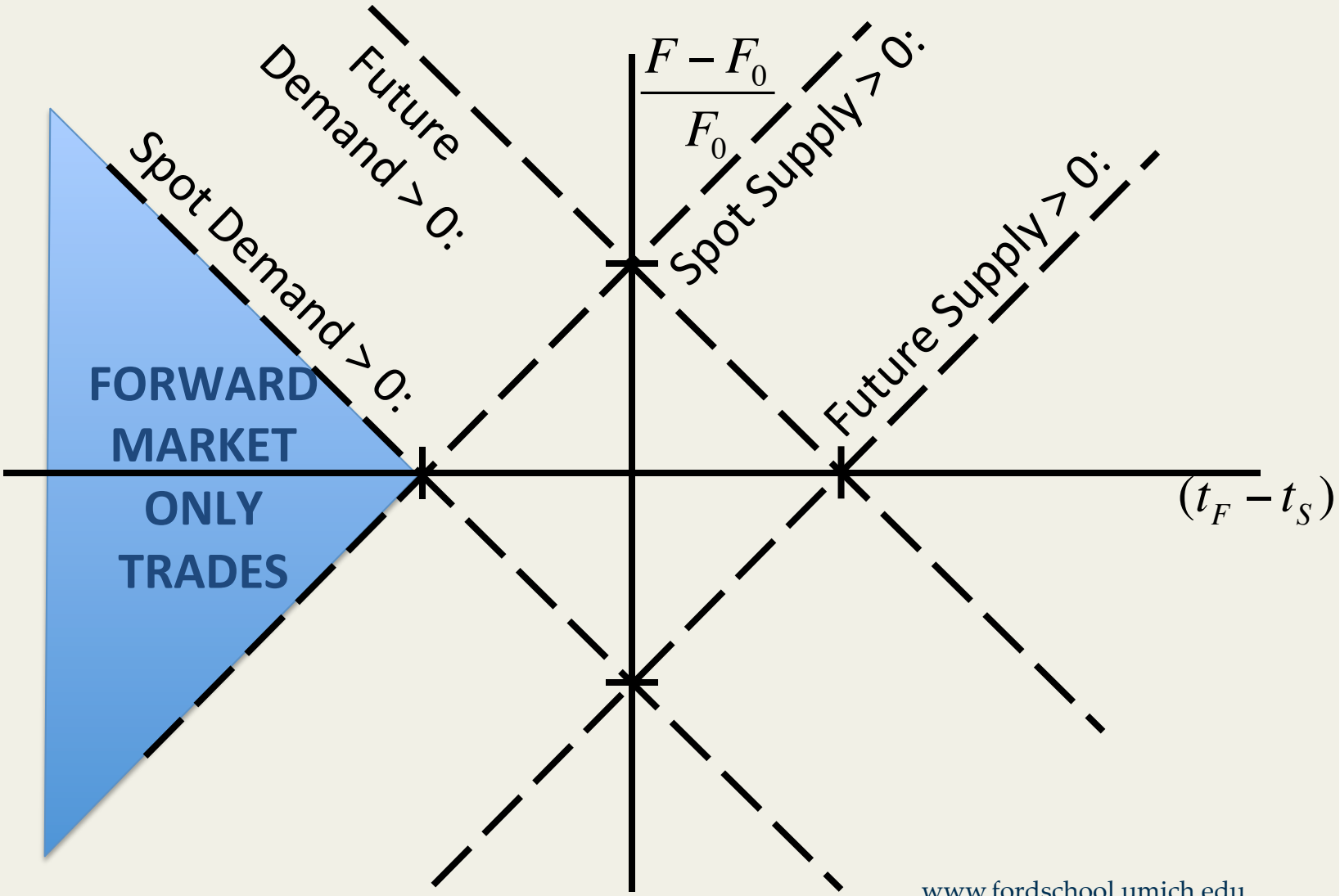
In combination:



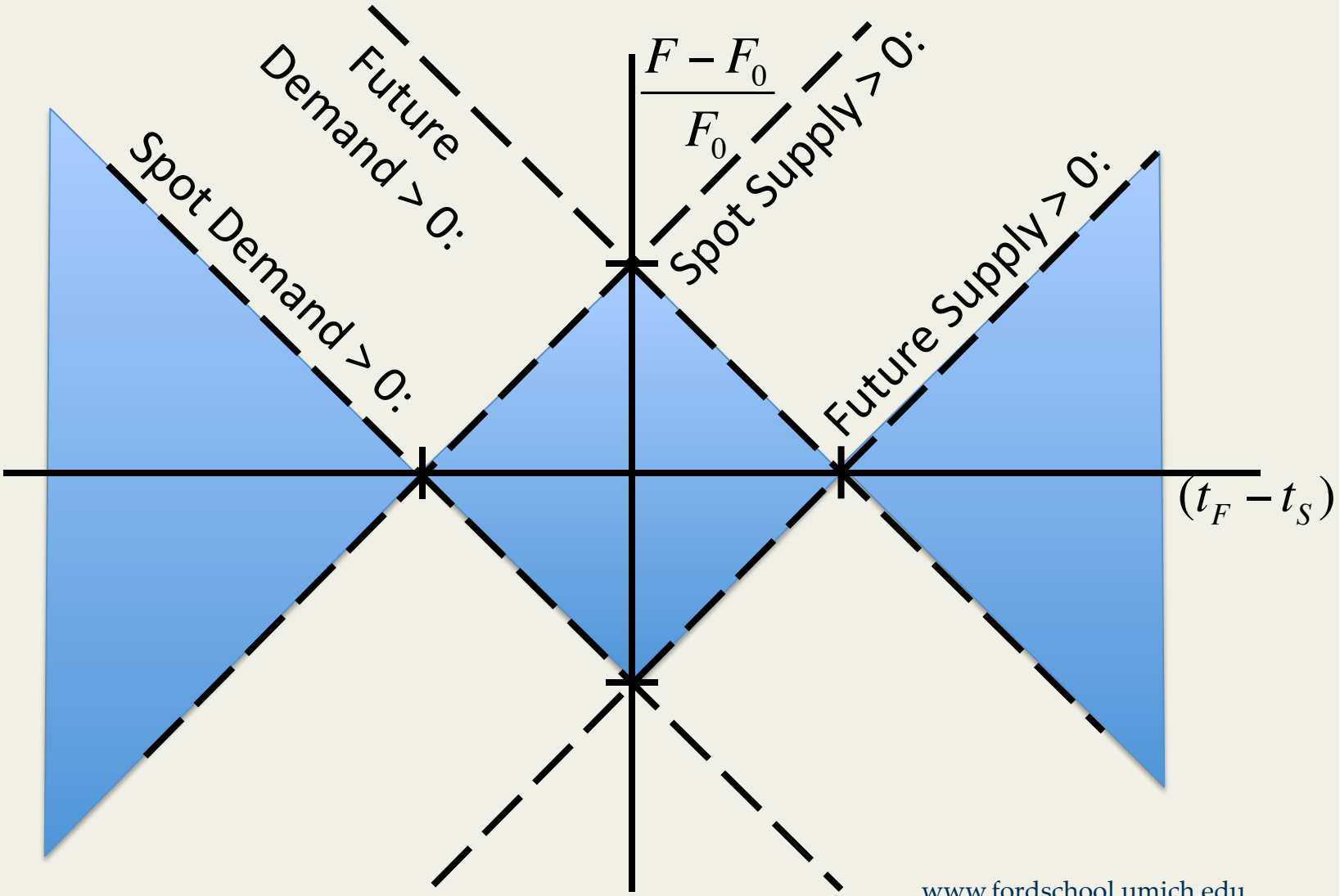
In combination:



In combination:



# EQUILIBRIA:



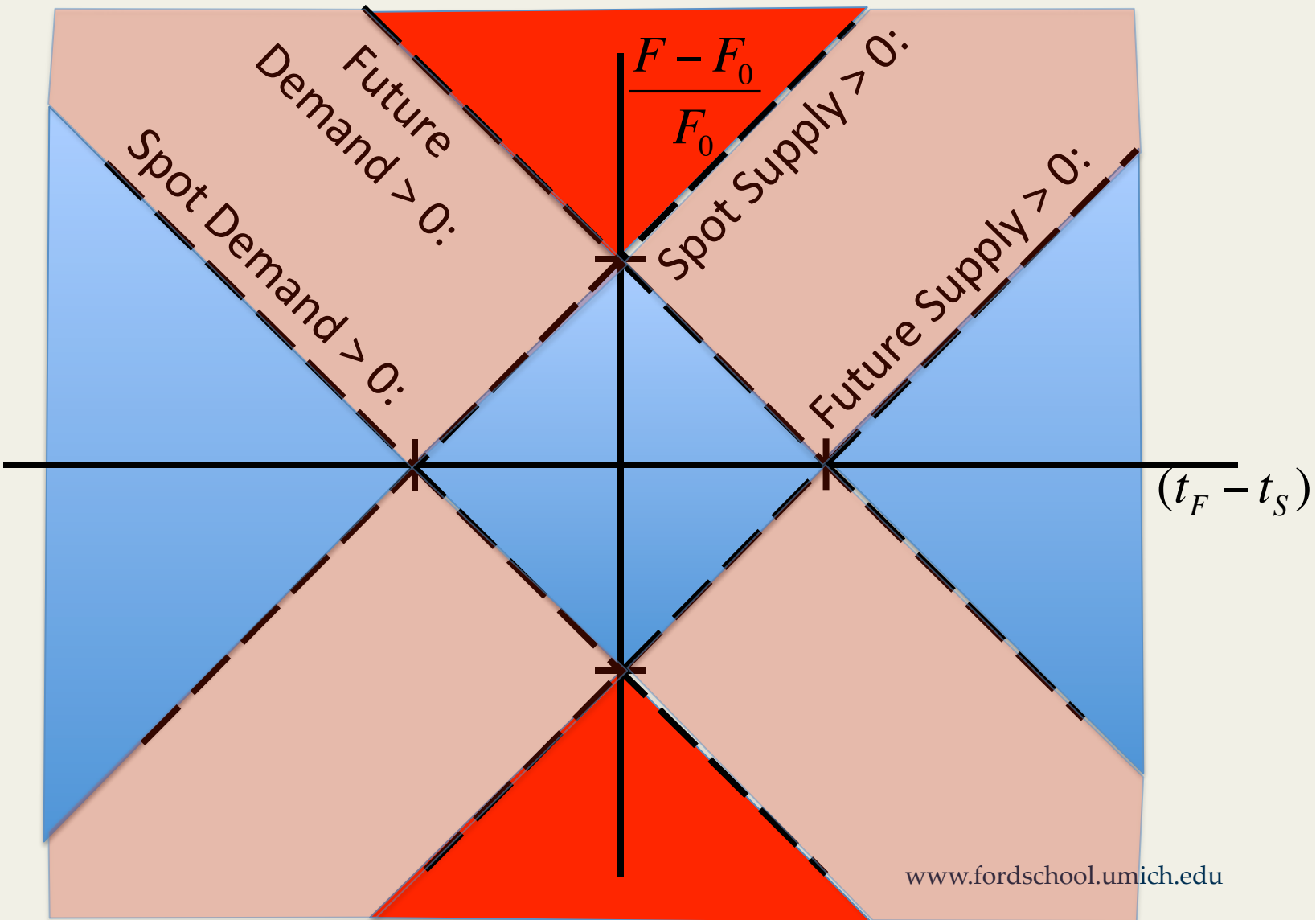
# Constraints Imposed by OWA

- A necessary (but not sufficient) condition for equilibrium is given in the paper as

$$\left| \frac{F - F_0}{F_0} \right| \leq (t + t^*) + |t_F - t_S|$$

- This excludes the dark-red shaded areas as follows...

# DISEQUILIBRIA:





# Compare Constraints Imposed by OWA and CIA

- OWA:  $\left| \frac{F - F_0}{F_0} \right| \leq (t + t^*) + |t_F - t_S| = K_{OWA}$

- CIA:  $\left| \frac{F - F_0}{F_0} \right| \leq t + t^* + t_S + t_F = K_{CIA}$

- If  $t_S, t_F > 0$ , then  $K_{OWA} < K_{CIA}$

- Thus OWA will prevent rates ever departing as far from interest parity as CIA would permit

# Stronger Constraint if Trade in Both Markets

- Recall the smaller area in the figure where there is positive trade in both markets. This requires the following stronger constraint:

$$\left| \frac{F - F_0}{F_0} \right| \leq (t + t^*) - |t_F - t_S|$$

- Note however that this is relevant only to the interbank market, where trades ultimately take place.
  - Retail clients may place orders with banks to both buy and sell on both markets, while the banks themselves use, say, only the spot and securities markets to effect the transactions.
  - The fact that forward exchange rates are quoted does not mean that the banks are necessarily engaging in forward contracts among themselves.

# Implications of the Results

- Deviations from covered interest parity will not be as large as predicted by Covered Interest Arbitrage.
- Covered Interest Arbitrage will not happen.
- The “Paradox” of Perfect Arbitrage
  - With CIA, students wonder how equilibrium can be sustained by CIA transactions that make no profit.
  - But OWA does not require profit, only cost minimization by market participants.

# Implications of the Results

- Measurement of Exchange Market Performance
  - Frenkel and Levich (1975) measured transactions costs in all 4 markets and tested the CIA prediction.
  - Better to have tested the OWA prediction.
  - Using their estimates of transaction costs, the two become:

$$K_{CIA} \geq 0.145\%$$

$$K_{OWA} \leq 0.044\%$$

The latter is less than 1/3 as large, so it may matter.

# Implications of the Results

- Viability of the Forward Market

- Frenkel and Levich always found  $t_F > t_S$
- They provided three estimates of  $t_F - t_S$ , all from 1962-67 data:

$$t_F - t_S = 0.025\%, 0.010\%, \& 0.015\%$$

- And one estimate of  $t + t^*$ :  $t + t^* = 0.038\%$
- This suggests that, while

$$t + t^* - (t_F - t_S) \text{ is between } [0.013\%, 0.028\%]$$

- Their estimate of  $t_F = 0.070\%$ . It wouldn't need to rise much to turn this negative, and prevent transactions in the forward market.
- McKinnon (1976) notes just such an increase with flexible exchange rates, and indeed use of the forward market was limited.

# Caveats

- Other considerations that could limit arbitrage of both kinds
  - Arbitrage funds may be limited (Keynes 1923)
  - Interest rates may depend on borrower's balance sheet (Prachowny 19970 & Frenkel 1973)
  - Assets provide convenience yield, so interest rate is not the only thing that matters (Tsiang 1959)
  - Political risk exists and differs across countries (Aliber 1973)

# Caveats

- All of these caveats may weaken the ability of both CIA and OWA to restrict deviations from interest parity.
  - They are more likely when arbitrageurs have substantial uncompleted contracts, as is likely during “turbulent periods”. Thus, as Frenkel & Levich (1977) find, it makes sense that violations of our conditions are then more likely.
  - But note that all these caveats apply to both CIA and OWA, and they do not therefore undermine the prediction that CIA is unlikely to occur.